Journal of Engineering Physics and Thermophysics, Vol. 81, No. 2, 2008

BRITTLE-FRACTURE CRITERION AND DURABILITY OF MATERIALS UNDER THERMOMECHANICAL ACTION

V. V. Shevelev

UDC 539.3

Consideration has been give to the thermoelasticity problem for a plane containing a rectilinear crack under the conditions of constant tensile stress and a nonuniform stationary temperature field. The system's thermodynamic potential from whose stationary condition the criterion of brittle fracture of a material has been established has been computed by solution of the problem indicated. The average durability of the material under thermomechanical action has been evaluated.

Introduction. Among the external factors acting on different materials under the actual conditions of their use, mechanical (load) and temperature (thermal action) fields are some of the most important. The reason is that when the action on the material is fairly long, the above fields finally lead to its fracture and, consequently, to loss of the necessary consumer properties of the product.

The destructive action of mechanical and temperature fields on glass products is based on the regions of higher-than-average stress which are created by them near structural defects (cracks as a rule) and the thermofluctuation character of destruction processes [1–3]. In the above region, this results in the reduction in energy barriers overcome by a system in rupture of loaded bonds and in the increase in barriers necessary for restoring them.

The structural-statistical kinetic theory of brittle fracture of materials that allows for both the random (i.e., stochastic) character of the initial distribution of structural defects by the degree of hazard and the stochastic character of the process of fracture itself caused by the thermofluctuation development of cracks has been elaborated upon in [3, 4].

It is noteworthy that brittle fracture is the most hazardous form of destruction, since it is sudden and has no warning signs. We note that this form of destruction is the most characteristic of glasses.

Formulation of the Problem. It is of practical interest to investigate the process of brittle fracture in the region of external action in which, from the viewpoint of the mechanics of a deformable body, we have no fracture no matter how long the external action may be (region of weak external action).

In actual practice, within the framework of the kinetic approach, fracture of a material, as has been shown in [3], is also possible in the region of weak external action (for which the asymptotic average-durability formula has been established in [3]) by virtue of the thermofluctuation character of the rupture of bonds. The boundary dividing the regions of weak and strong external actions is the well-known Griffith criterion which, according to [3, 4], determines the state of dynamic equilibrium for the fracture crack and coincides with the condition of stationary of the system's thermodynamic potential $\Delta\Phi$.

As applied to the process of fracture of the material, the Griffith criterion determines the regions of active ($\Delta\Phi$ decreases) and passive ($\Delta\Phi$ grows) fracture. Therefore, the primary problem in the prediction of the durability and strength of materials is in finding the system's thermodynamic potential by solution of the corresponding problem of elasticity theory. As far as we know, such an approach has been realized only in the case of pure mechanical action [3, 4]; in the case of thermomechanical action we believe that both the fracture criterion (in the form of a Griffith-type criterion) and the durability of materials have not been adequately investigated. A number of results based on finding the stress-intensity factors within the framework of the force approach in the case of thermomechanical action have been obtained in [5, 6].

Therefore, in this work, by solution of the thermoelastic problem for a sample with a fracture crack in the form of an internal rectilinear crack, we have obtained an expression for the sample's thermodynamic potential $\Delta\Phi$ and

M. V. Lomonosov Moscow State Academy of Fine Chemical Technology, 86 Prospekt Vernadskogo, Moscow, 119571, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 81, No. 2, pp. 394–400, March–April, 2008. Original article submitted June 27, 2007.

thereafter, from the $\Delta\Phi$ stationarity condition, have found the criterion of thermoelastic brittle fracture of the material and have evaluated its average durability. The system's thermodynamic potential Φ without allowance for the surface energy is determined by the relation [7]

$$\Phi = \iiint_V (f(\sigma, T) - \sigma_{ik}\varepsilon_{ik})\, dV , \tag{1}$$

where

$$f(\sigma, T) = f_0(T) - K\alpha(T - T_0) + \mu\left(\varepsilon_{ik} - \frac{1}{3}\delta_{ik}\varepsilon_{ll}\right)^2 + \frac{1}{2}K\varepsilon_{ll}^2 . \tag{2}$$

Allowing for the fact that $\sigma_{ik} = -K\alpha(T - T_0)\delta_{ik} + K\varepsilon_{ll}\delta_{ik} + 2\mu\left(\varepsilon_{ik} - \frac{1}{3}\delta_{ik}\varepsilon_{ll}\right)$ and, as a consequence, $\sigma_{ik}\varepsilon_{ik} = -K\alpha\varepsilon_{ll}(T - T_0) + K\varepsilon_{ll}^2 + 2\mu\left(\varepsilon_{ik}^2 - \frac{1}{3}\varepsilon_{ll}^2\right)$, expression (2) can be represented in the form

$$f(\sigma, T) = f_0(T) - \frac{1}{2}\sigma_{ik}\varepsilon_{ik} - \frac{1}{2}K\alpha\varepsilon_{ll}(T - T_0) . \tag{3}$$

Substituting (3) into (2), we obtain

$$\Phi = \iiint_V f_0(T)\, dV - \frac{1}{2}\iiint_V \sigma_{ik}\varepsilon_{ik}\, dV - \frac{1}{2}K\alpha\iiint_V (T - T_0)\,\varepsilon_{ll} dV . \tag{4}$$

Let us subtract from both sides of equality (4) the value of the thermodynamic potential $\Phi^{(1)}$ corresponding to the same stressed state of the sample but in absence of a crack. Then the increment $\Delta\Phi$ can be represented as

$$\Delta\Phi = -\frac{1}{2}\iiint_V \sigma_{ik}\varepsilon_{ik}\, dV + \frac{1}{2}\iiint_V \sigma_{ik}^{(1)}\varepsilon_{ik}^{(1)} dV - \frac{1}{2}K\alpha\iiint_V \varepsilon_{ll}(T - T_0)\, dV + \frac{1}{2}K\alpha\iiint_V \varepsilon_{ll}^{(1)}(T^{(1)} - T_0)\, dV . \tag{5}$$

In what follows the quantities referring to the stressed sample without a crack are marked with superscript 1, and those with a crack are marked with 2. We represent the stress and strain tensors and the displacement vector in the form

$$\sigma_{ik} = \sigma_{ik}^{(1)} + \sigma_{ik}^{(2)} , \tag{6}$$

$$\varepsilon_{ik} = \varepsilon_{ik}^{(1)} + \varepsilon_{ik}^{(2)} , \tag{7}$$

$$U_i = U_i^{(1)} + U_i^{(2)} . \tag{8}$$

On the exterior surface of the sample S_0, the conditions

$$\sigma_{ik}^{(1)} n_k = f_i , \quad \sigma_{ik}^{(2)} n_k = 0 , \quad i = 1, 2, 3 \tag{9}$$

are observed, whereas on the surface of the crack S_c considered as a cut of zero thickness in mechanics, we have the conditions

$$\left(\sigma_{ik}^{(1)} + \sigma_{ik}^{(2)}\right) n_k = f_{ic} , \quad U_i^{(1)} = 0 , \quad i = 1, 2, 3 . \tag{10}$$

Next, we take into account that, from the above expression for the quantity σ_{ik}, we obtain

$$\sigma_{ik}^{(1)}\varepsilon_{ik}^{(2)} = \sigma_{ik}^{(2)}\varepsilon_{ik}^{(1)} + K\alpha\left(T - T^{(1)}\right)\varepsilon_{ll}^{(1)} - \left(T^{(1)} - T_0\right)\varepsilon_{ll}^{(2)} .$$

As a result, $\Delta\Phi$ with allowance for the surface energy can now be represented in the form

$$\Delta\Phi = -\frac{1}{2}\iiint\limits_V \left(\sigma_{ik}^{(2)}\varepsilon_{ik}^{(2)} + 2\sigma_{ik}^{(2)}\varepsilon_{ik}^{(1)}\right)dV + \iint\limits_{S_c} \alpha_s dS$$

$$- K\alpha \iiint\limits_V \left(T - T^{(1)}\right)\operatorname{div}\mathbf{U}^{(1)}dV - \frac{1}{2}K\alpha \iiint\limits_V \left(T - T^{(1)}\right)\operatorname{div}\mathbf{U}^{(2)}dV . \tag{11}$$

Transforming the first integral in (11) to the surface one and allowing for the fact that, in accordance with the second conditions of (9) and (10), we have $\iiint\limits_V \sigma_{ik}^{(2)}\varepsilon_{ik}^{(2)}dV = 0$, we obtain, taking into account the first conditions of (9) and (10):

$$\Delta\Phi = \frac{1}{2}\iint\limits_{S_c} \sigma_{ik}^{(1)} n_k U_i^{(2)} dS - \frac{1}{2}\iint\limits_{S_c} f_{ic} U_i^{(2)} dS + \iint\limits_{S_c} \alpha_s dS$$

$$- \frac{1}{1 - 2\nu} E\alpha_T \left(\frac{1}{2}\iiint\limits_V \left(T - T^{(1)}\right)\operatorname{div}\mathbf{U}^{(2)}dV + \iiint\limits_V \left(T - T^{(1)}\right)\operatorname{div}\mathbf{U}^{(1)}dV\right). \tag{12}$$

To find $\Delta\Phi$ from the general expression (12) we must completely prescribe the thermomechanical action on the material, including the orientation of the crack. Cracks oriented perpendicularly to the external tensile stress σ are the most hazardous, since in this case the local stress in the crack tip is the largest. Therefore, in what follows we restrict our consideration to a sample containing an internal rectilinear crack of length l that is oriented perpendicularly to the external tensile stress σ, extending the sample along the Oy axis in the stationary temperature field $T(x, y)$. Then, in the absence of the crack, the stress field in the material is determined by just one component of the stress tensor, namely $\sigma_{yy}^{(1)} = \sigma$ (in what follows the value of the subscripts, equal to unity, corresponds to x, and the value equal to two corresponds to y). We take the temperature field in the absence of the crack equal to T_0, $T^{(1)}(x, y) = T_0$. The crack itself is located in the $y = 0$ plane and is interpreted as a zero-thickness cut located within $|x| \le l/2$. At the cut edges, the temperature is constant and equal to T_1. The above thermomechanical action on the material initiates, in it, temperature and strain fields possessing the following symmetry:

$$T(-x, y) = T(x, y) = T(x, -y) , \tag{13}$$

$$U_y^{(2)}(-x, y) = U_y^{(2)}(x, y) , \tag{14}$$

$$U_y^{(2)}(x, -y) = -U_y^{(2)}(x, y) , \tag{15}$$

$$\operatorname{div}\mathbf{U}^{(2)}(x, -y) = \operatorname{div}\mathbf{U}^{(2)}(x, y) . \tag{16}$$

Allowing for the two-dimensional character of the thermodynamic action on the material and for symmetry conditions (13)–(16), from the general expression (12) for the quantity $\Delta\Phi$, under the specified conditions of external action, we obtain the following expression for the change in the thermodynamic potential:

$$\Delta\Phi = -2\sigma \int_0^{l/2} U_y^{(2)}(x, 0)\, dx - \frac{2E\alpha_T}{1 - 2\nu}\int_0^{\infty} dx \int_0^{\infty} W(x, y)\operatorname{div}\mathbf{U}^{(2)}(x, y)\, dy$$

$$- 4\alpha_T \sigma \int_0^{\infty} dx \int_0^{\infty} W(x, y)\, dy + 2\alpha_s l\,. \tag{17}$$

In deriving (17), we have allowed for the fact that $\sigma_{ik}^{(1)} n_k U_i^{(2)} = -\sigma U_y^{(2)}(x, 0)$ at the upper and lower cut edges; for the plane stress state, we have div $\mathbf{U}^{(1)}(x, y) = (1 - 2\nu)\sigma/E$. We restrict ourselves to the case where forces at the crack edges do not act, i.e., $f_{ic} = 0$.

As is seen from (17), to determine $\Delta\Phi$ we must find the functions div $\mathbf{U}^{(2)}(x, y)$, $W(x, y)$, and $U_y^{(2)}(x, 0)$. In the case of the plane stress state we must first find the stress field $\sigma_{ik}^{(2)}$ (i and k = 1 and 2). This can be done using Fourier transformation in variable x. Applying this transformation to the equilibrium equations $\partial\sigma_{ik}^{(2)}/\partial x_k = 0$, i = 1 and 2 (in what follows we replace the value of the subscript $i = 1$ by x and $i = 2$ by y) with the boundary conditions

$$\sigma_{yy}^{(2)}(x, 0) = -\sigma\,, \quad |x| \le \frac{l}{2}\,, \tag{18}$$

$$U_y^{(2)}(x, 0) = 0\,, \quad |x| \le \frac{l}{2}\,, \tag{19}$$

$$\sigma_{xy}^{(2)}(x, 0) = 0\,, \quad x \in \mathbb{R}\,, \tag{20}$$

leads to the following expressions:

$$\sigma_{yy}^{(2)}(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-|\omega|\,|y| - i\omega x)\, A(\omega)\, (1 + |\omega|\,|y|)\, d\omega\,, \tag{21}$$

$$\sigma_{xx}^{(2)}(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-|\omega|\,|y| - i\omega x)\, A(\omega)\, (1 - |\omega|\,|y|)\, d\omega\,, \tag{22}$$

$$U_y^{(2)}(x, y) = \frac{-1}{\sqrt{2\pi}\, E} \int_{-\infty}^{\infty} \exp(-|\omega|\,|y| - i\omega x)\, A(\omega)\, (2 \operatorname{sign} y + (1 + \nu)\, |\omega|\, y)\, \frac{d\omega}{|\omega|} - \alpha_T \int_y^{\infty} W(x, \eta)\, d\eta\,. \tag{23}$$

The function $W(x, y)$ involved in (23) for $y > 0$ is the solution of the boundary-value problem

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = 0\,, \quad x \in \mathbb{R}\,, \quad y > 0\,; \tag{24}$$

$$W(x, 0) = W_1\,, \quad |x| < \frac{l}{2}\,; \tag{25}$$

$$\frac{\partial W(x, 0)}{\partial y} = 0\,, \quad |x| > \frac{l}{2}\,; \tag{26}$$

$$\lim_{\sqrt{x^2+y^2} \to \infty} W(x, y) = 0\,. \tag{27}$$

The solution of problem (24)–(27) using Fourier transformation can be represented in the form (also true for $y < 0$)

$$W(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-i\omega x - |\omega|\,|y|)\, B(\omega)\, d\omega . \tag{28}$$

The unknown functions $A(\omega)$ and $B(\omega)$ are found from conditions (18), (19), (25), and (26) respectively. This leads us to the necessity of solving the following dual integral equations and of allowing for the fact that $A(-\omega) = A(\omega)$ and $B(-\omega) = B(\omega)$ in accordance with symmetry conditions (13)–(16):

$$\int_0^{\infty} A(\omega) \cos(\omega x)\, d\omega = -\sigma \sqrt{\frac{\pi}{2}}, \quad 0 < x < \frac{l}{2}; \tag{29}$$

$$\int_0^{\infty} \Big(2A(\omega) + \alpha_T E B(\omega)\Big) \cos(\omega x) \frac{d\omega}{\omega} = 0, \quad x > \frac{l}{2}; \tag{30}$$

$$\int_0^{\infty} B(\omega) \cos(\omega x)\, d\omega = W_1 \sqrt{\frac{\pi}{2}}, \quad 0 < x < \frac{l}{2}; \tag{31}$$

$$\int_0^{\infty} B(\omega) \cos(\omega x)\, d\omega = 0, \quad x > \frac{l}{2}. \tag{32}$$

We obtain the solution of these equations using the tables of dual integral equations [4]:

$$A(\omega) = -\frac{l}{2}\sqrt{\frac{\pi}{2}}\, \sigma \left| J_1\left(\frac{\omega l}{2}\right) \right|, \quad B(\omega) = \frac{l}{2}\sqrt{\frac{\pi}{2}}\, W_1 \left| J_1\left(\frac{\omega l}{2}\right) \right| . \tag{33}$$

With account for (33), from (23), we find (using the tables of [8]) $U_y^{(2)}(x, 0)$:

$$U_y^{(2)}(x, 0) = \frac{l}{E} \int_0^{\infty} (2\sigma - \alpha_T E W_1)\, J_1\left(\frac{\omega l}{2}\right) \cos(\omega x) \frac{d\omega}{\omega} \Rightarrow U_y^{(2)} = \left(\frac{2\sigma}{E} - \alpha_T W_1\right) \sqrt{\frac{l^2}{4} - x^2}, \quad |x| < \frac{l}{2}; \tag{34}$$

$$\int_0^{l/2} U_y^{(2)}(x, 0)\, dx = \left(\frac{2\sigma}{E} - \alpha_T W_1\right) \frac{l^2 \pi}{16} . \tag{35}$$

To compute the remaining integrals in (17) we use the Prandtl formula [9]. We allow for the fact that, for the plane stress state, we obtain

$$\operatorname{div} \mathbf{U}^{(2)} = \frac{1 - 2\nu}{E} \left(\sigma_{xx}^{(2)} + \sigma_{yy}^{(2)}\right) + 3\alpha_T W(x, y) .$$

Combining (21), (22), and (23) with the corresponding factors, we obtain with allowance for the evenness of $A(\omega)$ and $B(\omega)$:

$$\operatorname{div}\mathbf{U}^{(2)} = \sqrt{\frac{2}{\pi}}\int_0^\infty \cos(\omega x)\exp(-\omega\,|y|)\left(\frac{2(1-2\nu)}{E}A(\omega) + 3\alpha_T B(\omega)\right)d\omega .$$

Substituting this expression and (28) into (17), we find

$$\Delta\Phi = \left(-\frac{\pi\sigma l^2}{8}\left(\frac{2\sigma}{E} - \alpha_T W_1\right) - \frac{2E\alpha_T}{1-2\nu}\int_0^\infty dx\int_0^\infty W(x, y)\,dy\sqrt{\frac{2}{\pi}}\int_0^\infty \cos(\omega x)\left(\frac{2(1-2\nu)}{E}A(\omega) + 3\alpha_T B(\omega)\right)d\omega\right.$$

$$\left. - 4\alpha_T\sigma\sqrt{\frac{2}{\pi}}\int_0^\infty dx\int_0^\infty B(\omega)\cos(\omega x)\,d\omega\int_0^\infty \exp(-\omega y)\,dy + 2\alpha_s l\right)\lambda_\pi .$$

Changing the order of integration with respect to x and ω in the second term and performing integration with respect to y, we have

$$\Delta\Phi = \left(-\frac{\pi\sigma l^2}{8}\left(\frac{2\sigma}{E} - \alpha_T W_1\right) - \frac{E\alpha_T}{1-2\nu}\int_0^\infty B(\omega)\left(\frac{2(1-2\nu)}{E}A(\omega) + 3\alpha_T B(\omega)\right)\frac{d\omega}{\omega}\right.$$

$$\left. - 4\alpha_T\sigma\int_0^\infty dx\int_0^\infty B(\omega)\cos(\omega x)\frac{d\omega}{\omega} + 2\alpha_s l\right)\lambda_\pi .$$

Substituting the expressions $A(\omega)$ and $B(\omega)$ determined by formulas (33) and (34) into the resulting formula and performing integration, we obtain

$$\Delta\Phi = \left(-\frac{\pi\sigma^2 l^2}{4E} - \frac{3\pi E(\alpha_T W_1 l)^2}{16(1-2\nu)} + 2\alpha_s l\right)\lambda_\pi . \tag{36}$$

Discussion of Results. We find the extremum of $\Delta\Phi$ as a function of the initial crack length l. Differentiation of $\Delta\Phi$ with respect to l yields

$$\frac{d\Delta\Phi}{dl} = \left(\frac{-\pi\sigma^2 l}{2E} - \frac{3\pi E(\alpha_T W_1)^2 l}{8(1-2\nu)} + 2\alpha_s\right)\lambda_\pi . \tag{37}$$

Equating the derivative to zero, we find the critical crack length l^* determining the state of dynamic equilibrium of the crack, i.e., the beginning of its directed growth:

$$l^* = \frac{4\alpha_s}{\dfrac{\pi\sigma^2}{E} + \dfrac{3\pi E(\alpha_T W_1)^2}{4(1-2\nu)}} . \tag{38}$$

From formula (38), we obtain the critical crack length in the case of pure mechanical $l^* = \dfrac{4\alpha_s E}{\pi\sigma^2}$ and pure temperature $l^* = \dfrac{16\alpha_s(1-2\nu)}{3\pi E(\alpha_T W_1)^2}$ action. Formula (38) points to the upper permissible limit of the initial length l of internal cracks in the sample. The observance of the condition $l_{max} < l^*$ ensures its fairly long average durability τ, which is determined in this case, according to [3], by the expression

$$\tau \approx \frac{1}{\lambda w^+ (l^*)} \exp\left(\frac{\Delta\Phi (l^*)}{kT}\right) \sqrt{\frac{2\pi kT}{|\Delta\Phi'' (l^*)|}} \ , \tag{39}$$

where $w^+(l) = \nu_0 \exp\left(-\frac{U - V_f \chi\sigma\sqrt{l/\lambda}}{kT}\right)$; $\chi = 1.12$. Computing the quantities involved in (39) we find the average durability of the material in the region of weak thermomechanical action:

$$\tau \approx \frac{1}{\lambda w^+ (l^*)} \exp\left(\frac{\lambda_\pi \alpha_s l^*}{kT}\right) \sqrt{\frac{\pi kT l^*}{\alpha_s \lambda_\pi}} \ . \tag{40}$$

From the condition of $\Delta\Phi$ stationarity, we can also find an expression for the threshold of the external stress σ^* beginning with which we have the stage of active growth of a crack of initial dimension l_0 for $\sigma > \sigma^*$:

$$\sigma^* = \sqrt{\frac{4\alpha_s E}{\pi l_0} - \frac{3\pi (\alpha_T E W_1)^2}{4\pi (1 - 2\nu)}} \ . \tag{41}$$

When $W_1 = 0$ we obtain from (41) the well-known result, the so-called Griffith criterion: $\sigma^* = 2\sqrt{\frac{\alpha_s E}{\pi l_0}}$, and for $\sigma = 0$ we obtain the criterion of pure thermal damage

$$W_1^* = \pm \frac{4}{\alpha_T} \sqrt{\frac{\alpha_s (1 - 2\nu)}{\pi l_0}} \ . \tag{42}$$

The crack of length l_0 has the average growth rate $v_c > 0$ when $|W_1| > |W_1^*|$ and $v_c < 0$ when $|W_1| < |W_1^*|$ but we can have a thermofluctuation growth up to a dimension for which $v_c > 0$ now. Also, (41) shows that for $l_0 > l_{cr}$, where $l_{cr} = \frac{16\alpha_s(1 - 2\nu)}{3\pi E(\alpha_T W_1)^2}$, the crack begins to grow without the external mechanical action.

CONCLUSIONS

1. We have obtained the expression for the increment in the thermodynamic potential of the material $\Delta\Phi$ in the case of thermal and mechanical action on it.

2. The stationarity of $\Delta\Phi$ yields the conditions for active and passive development of fracture and the asymptotic formula for the average durability of the material.

NOTATION

E, Young modulus, N/m^2; f_i and f_{ic}, components of the external forces $\bar{f}$ and $\bar{f}_c$ acting per unit area of the body's surface and the crack surface respectively, N/m^2; $f(\sigma, T)$, free energy of a unit volume of the material, J/m^3; $f_0(T)$, free energy of a unit volume of an unstrained material, J/m^3; $J_1(x)$, cylindrical function of the first kind; i and k, tensor indices; K, modulus of dilatation, N/m^2; k, Boltzmann constant, J/K; l, crack length, m; l_0, initial crack length, m; l_{cr}, critical crack length, m; l_{max}, maximum length of the internal cracks in the material, m; l^*, length of a crack in the state of dynamic equilibrium, m; S_0, exterior sample surface, m^2; S_c, crack surface, m^2; sign y, sign function, sign y

$= \begin{cases} -1, & y < 0; \\ 0, & y = 0; \\ 1, & y > 0; \end{cases}$ $T(x, y)$, temperature at a point with rectangular Cartesian coordinates (x, y), K; T_0, temperature at which the system in question, in the absence of external forces, is assumed to be unstrained, K; T_1, temperature on the crack surface, K; $T^{(1)}(x, y)$, temperature field in the absence of a crack, K; **U**, displacement vector, m; U, activation energy of bond rupture for $\sigma = 0$, J; U_i, ith component of the displacement vector; $i = 1$, 2, and 3 or respectively x, y, and z; V, body's volume, m^3; V_f, fluctuation volume, m^3; v_c, rate of growth of the crack, m/sec; $W(x, y) = T - T^{(1)}$, K; $W_1(x, y) = T_1 - T_0$, K; W_1^*, temperature difference determining the beginning of active growth of the crack in the absence of stress, K; $w^+(l)$, frequency of bond rupture in the tip of the crack of length l, sec^{-1}; α and $\alpha_T = \alpha/3$, coefficients of volume and linear thermal expansion of the material respectively, deg^{-1}; α_s, specific free surface energy, J/m^2; δ_{ik}, Kronecker symbol; $\delta_{ik} = 1$ if $i = k$ and $\delta_{ik} = 0$ if $i \neq k$, $i = 1$, 2, and 3; $k = 1$, 2, and 3; Φ, thermodynamic potential of the system, J; $\Delta\Phi$, increment in the system's thermodynamic potential, J; ε_{ik}, strain tensor; λ_π, crack-front length involved in the crack's fluctuation movement, m; λ, characteristic distance over which the crack moves in single fluctuation of the bond rupture, m; μ, shear modulus, N/m^2; ν, Poisson coefficient; ν_0, frequency of attempts to overcome the energy barrier on the path of bond rupture, sec^{-1}; σ, tensile external stress, N/m; σ_{ik}, stress tensor, N/m^2; σ^*, external stress determining the beginning of active growth of the crack, N/m^2; τ, average durability of the material, sec. Subscripts: cr, critical; s, surface; c, crack; f, fluctuation; max, maximum.

REFERENCES

1. G. M. Bartenev, *Strength and Destruction of Polymers* [in Russian], Khimiya, Moscow (1984).
2. V. R. Regel, A. I. Slutsker, É. E. Tomashevskii, *The Kinetic Nature of the Strength of Solid Bodies* [in Russian], Nauka, Moscow (1974).
3. V. V. Shevelev and É. M. Kartashov, Some static aspects of brittle fracture and durability of polymers. Crack-bearing materials, *Vysokomolek. Soed.*, **39B**, No. 2, 371–381 (1977).
4. É. M. Kartashov, B. Tsoi, and V. V. Shevelev, *Structural-Statistical Kinetics of the Destruction of Polymers* [in Russian], Khimiya, Moscow (2002).
5. É. M. Kartashov, Coefficients of the intensity of stresses in an infinite crack-bearing plate in an inhomogeneous stationary temperature field, *Fizika*, No. 3, 7–13 (1979).
6. É. M. Kartashov, V. V. Shevelev, and A. A. Valishin, Time dependence of the strength of brittle polymers on nonisothermal loading, *Dokl. Ross. Akad. Nauk*, **350**, No. 2, 216–219 (1996).
7. L. D. Landau and E. M. Lifshits, *Elasticity Theory* [in Russian], Nauka, Moscow (1987).
8. I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals, Sums, Series, and Products* [in Russian], Nauka, Moscow (1971).
9. E. C. Titchmarsh, *Introduction to the Theory of Fourier Integrals* [Russian translation], OGIZ, Moscow–Leningrad (1948).